Matrices for $\vec{\alpha}$ and β :-

The squares of all the four matrices are unity so that that their eigen values are +1 and -1 where

$$\alpha_x = \begin{bmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{bmatrix} \text{ where } \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

[$\sigma_x, \sigma_y, \sigma_z$ = Pauli's matrices]

$$\alpha_x = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\alpha_y = \alpha_y = \begin{bmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{bmatrix} \text{ where } \sigma_y = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

$$\alpha_y = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}$$

$$\alpha_z = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix} \quad \text{where} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\alpha_z = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

These are 4x4 matrices are evidently Hermitian and in abbreviate form may be expressed as :-

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} ; \quad \vec{\alpha} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix}$$

and the Dirac func^n ψ is given by

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}$$

From schroedinger eq^n

$$H\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t}\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = \begin{bmatrix} \partial\psi_1/\partial t \\ \partial\psi_2/\partial t \\ \partial\psi_3/\partial t \\ \partial\psi_4/\partial t \end{bmatrix}$$

again from

$$H\psi = E\psi$$

$$(c\vec{\alpha}\cdot p + \beta mc^2)\psi = E\psi$$

$$(c\alpha_x p_x + c\alpha_y p_y + c\alpha_z p_z + \beta mc^2)\psi = E\psi \quad (11)$$

with $p_x = -i\hbar\, \partial/\partial x$ and so on, is called the Dirac eq^n for free particle

Now, substituting α's and β by specific matrices and replace ψ by four-component column symbol

$$\begin{bmatrix} mc^2 & 0 & cp_z & c(p_x - ip_y) \\ 0 & mc^2 & c(p_x + ip_y) & -cp_z \\ cp_z & c(p_x - ip_y) & -mc^2 & 0 \\ c(p_x + ip_y) & -cp_z & 0 & -mc^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = E \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}$$

So that this eq^n reduces to the four simultaneous eq^n

$$(mc^2)\psi_1 + cp_z\psi_3 + c(p_x - ip_y)\psi_4 = E\psi_1$$

$$(mc^2)\psi_2 + cp_z\psi_4 + c(p_x + ip_y)\psi_3 = E\psi_2$$

$$(-mc^2)\psi_3 + cp_z\psi_1 + c(p_x - ip_y)\psi_2 = E\psi_3$$

$$(-mc^2)\psi_4 + cp_z\psi_2 + c(p_x + ip_y)\psi_1 = E\psi_4$$

These equations may be expressed as

$$\left.\begin{aligned}
(E - mc^2)\psi_1 - cp_z\psi_3 - c(p_x - ip_y)\psi_4 &= 0\\
(E - mc^2)\psi_2 - c(p_x + ip_y)\psi_3 + cp_z\psi_4 &= 0\\
(E + mc^2)\psi_3 - cp_z\psi_1 - c(p_x - ip_y)\psi_2 &= 0\\
(E + mc^2)\psi_4 - c(p_x + ip_y)\psi_1 + cp_z\psi_2 &= 0
\end{aligned}\right\} \text{—(12)}$$

Finally we replace p_x by $-i\hbar\frac{\partial}{\partial x}$ etc. we get

$$\left.\begin{aligned}
&(E - mc^2)\psi_1 + i\hbar c\frac{\partial\psi_3}{\partial z} + i\hbar c\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_4 = 0\\
&(E - mc^2)\psi_2 + i\hbar c\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_3 - i\hbar c\frac{\partial\psi_4}{\partial z} = 0\\
&(E + mc^2)\psi_3 + i\hbar c\frac{\partial\psi_1}{\partial z} + i\hbar c\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_2 = 0\\
&(E + mc^2)\psi_4 + i\hbar c\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_1\\
&\qquad - i\hbar c\frac{\partial\psi_2}{\partial z} = 0
\end{aligned}\right\}$$